Hochschule RheinMain
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WS 2026/27

2. Homework Assignment Advanced Classical Mechanics

Exercise 1: We consider the differential equation (ODE, ordinary differential equation) for a damped harmonic oscillator with a time-dependent damping,

$$\ddot{x}_t \;+\; \gamma_t\, \dot{x}_t \;+\; \omega^2 x_t \;\;=\;\; 0 \tag{1}$$

Compared to the situation on the first exercise sheet, γ_t is now allowed to depend on the time t. Show that, using the ansatz

$$x_t \;\;=\;\; e^{-\frac{1}{2}\int_0^t \gamma_s\, ds}\, y_t$$

the ODE (1) can be reduced to the ODE

$$\ddot{y}_t \;+\; \omega_t^2\, y_t \;\;=\;\; 0 \tag{2}$$

with a time-dependent frequency ω_t given by

$$\omega_t^2 \;\;=\;\; \omega^2 \;-\; \tfrac{\gamma_t^2}{4} \;-\; \tfrac{\dot{\gamma}_t}{2} \;\;. \tag{3}$$

Exercise 2: Now consider specifically the ODE

$$\ddot{x}_t \;+\; 2\mu \sin(\mu t)\, \dot{x}_t \;+\; x_t \;\;=\;\; 0 \tag{4}$$

with, for example,

$$\mu \;\;=\;\; \tfrac{1}{50}$$

a) Write down the analytical expression for the damping factor $e^{-\frac{1}{2}\int_0^t \gamma_s\, ds}$ explicitly.

b) Write down the analytical expression for the time-dependent frequency

$$\omega_t^2 \;\;=\;\; \omega^2 \;-\; \tfrac{\gamma_t^2}{4} \;-\; \tfrac{\dot{\gamma}_t}{2}$$

explicitly.

c) Convince yourself that $\omega_t^2 \approx 1$ is a reasonable approximation, and under this assumption give an approximate solution of (4) in closed analytic form. To do so, choose the initial conditions $x_0 = 1$ and $\dot{x}_0 = 0$.

d) Plot the solution from (c) on the interval $[0, 200\pi] \;\approx\; [0, 628]$.

e) If you still have time and are motivated, you could try to check the quality of the approximation from c) with a numerical simulation. One obtains roughly the following picture:

