

1. Homework Assignment Advanced Classical Mechanics

Exercise 1 (harmonic oscillator): The harmonic oscillator is one of the few systems which can be completely solved analytically in closed form. Furthermore, many oscillating systems, for small displacements from their equilibrium position, can be approximated by a harmonic oscillator. This is basically due to the following calculation,

$$\begin{aligned} V(x) &= V(x_{\min}) + \underbrace{V'(x_{\min})}_{=0} (x - x_{\min}) + \frac{1}{2} V''(x_{\min}) (x - x_{\min})^2 + O((x - x_{\min})^3) \\ &\approx V(x_{\min}) + \frac{1}{2} V''(x_{\min}) (x - x_{\min})^2 \end{aligned} \quad (1)$$

where $x_{\min}$ denotes the equilibrium position and $V = V(x)$ is the potential which generates the force through $F = -\nabla V = -V'(x)$, let's stick to the one dimensional case. The potential in the last line of (1) is quadratic in x and hence is the potential of a harmonic oscillator.

- a) Consider the differential equation for a harmonic oscillator given by¹

$$\ddot{x}_t + \omega^2 x_t = 0 \quad (2)$$

Write down the general real solution with initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$.

- b) The differential equation of a damped harmonic oscillator is given by

$$\ddot{x}_t + \gamma \dot{x}_t + \omega^2 x_t = 0 \quad (3)$$

with some $\gamma > 0$. Write down the general real solution with initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$. To this end, put $\gamma = 2\mu$ and consider the cases $\mu < \omega$, $\mu = \omega$ and $\mu > \omega$.

- c) The differential equation of a forced and damped harmonic oscillator with excitation frequency ω_0 is given by

$$\ddot{x}_t + \gamma \dot{x}_t + \omega^2 x_t = a_0 \sin(\omega_0 t) \quad (4)$$

Thereby we assumed that at time $t = 0$ the exciting force vanishes such that on the right hand side we only have a sine but no cosine term. Again, write down the general real solution with initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$.

Due to the importance of the harmonic oscillator we do this assignment sheet in class in the next week2.pdf.

¹the x_t has the meaning of $x_t - x_{\min}$ in (1)