

## Lösungen zum 10. Übungsblatt Einführung in die Finanzmathematik

**Aufgabe 1:** Wir haben

$$\begin{aligned} V_0 &= e^{-rT} \int_{\mathbb{R}} H(S_T) e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} \int_{\mathbb{R}} H(S_0 e^{\sigma\sqrt{T}x + (r-\sigma^2/2)T}) e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \end{aligned}$$

Wegen

$$\begin{aligned} S_T &\geq K \\ \Leftrightarrow S_0 e^{\sigma\sqrt{T}x + (r-\sigma^2/2)T} &\geq K \\ \Leftrightarrow \sigma\sqrt{T}x + (r - \sigma^2/2)T &\geq \log[K/S_0] \\ \Leftrightarrow \sigma\sqrt{T}x &\geq \log[K/S_0] - (r - \sigma^2/2)T \\ \Leftrightarrow x &\geq \frac{\log[K/S_0] - (r - \sigma^2/2)T}{\sigma\sqrt{T}} \\ \Leftrightarrow x &\geq - \frac{\log[S_0/K] + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \\ \Leftrightarrow x &\geq -d_- \end{aligned}$$

bekommen wir

$$\begin{aligned} V_0 &= e^{-rT} \int_{\mathbb{R}} H(S_0 e^{\sigma\sqrt{T}x + (r-\sigma^2/2)T}) e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} \int_{-d_-}^{\infty} 1 \cdot e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} \int_{-\infty}^{d_-} e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} N(d_-) . \end{aligned}$$