Lösungen zum 10. Übungsblatt
Einführung in die Finanzmathematik

Aufgabe 1: Wir haben

$$\begin{aligned} V_0 &= e^{-rT} \int_{\mathbb{R}} H(S_T)\, e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} \int_{\mathbb{R}} H\big(S_0\, e^{\sigma\sqrt{T}\,x + (r-\sigma^2/2)T}\big)\, e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \end{aligned}$$

Wegen

$$\begin{aligned} & S_T \geq K \\ \Leftrightarrow\quad & S_0\, e^{\sigma\sqrt{T}\,x + (r-\sigma^2/2)T} \geq K \\ \Leftrightarrow\quad & \sigma\sqrt{T}\,x + (r-\sigma^2/2)T \geq \log[K/S_0] \\ \Leftrightarrow\quad & \sigma\sqrt{T}\,x \geq \log[K/S_0] - (r-\sigma^2/2)T \\ \Leftrightarrow\quad & x \geq \frac{\log[K/S_0] - (r-\sigma^2/2)T}{\sigma\sqrt{T}} \\ \Leftrightarrow\quad & x \geq -\frac{\log[S_0/K] + (r-\sigma^2/2)T}{\sigma\sqrt{T}} \\ \Leftrightarrow\quad & x \geq -d_- \end{aligned}$$

bekommen wir

$$\begin{aligned} V_0 &= e^{-rT} \int_{\mathbb{R}} H\big(S_0\, e^{\sigma\sqrt{T}\,x + (r-\sigma^2/2)T}\big)\, e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} \int_{-d_-}^{\infty} 1 \cdot e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} \int_{-\infty}^{d_-} e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}} \\ &= e^{-rT} N(d_-)\,. \end{aligned}$$