Hochschule RheinMain
Prof. Dr. D. Lehmann

SS 2025

Lösungen Übungsblatt 7
Finanzmathematik II

1.Aufgabe: Mit Teil (b) von Theorem 7.1 erhalten wir

a)

$$\begin{aligned}\mathsf{P}\big[\,x_T \le 1 \,\wedge\, \max_{t\in[0,T]} x_t \le 2\,\big] \;&=\; N\big(\tfrac{1}{\sqrt{1}}\big) + N\big(\tfrac{2\times 2-1}{\sqrt{1}}\big) - 1 \\ &=\; N(1) + N(3) - 1 \;\approx\; 0.84\end{aligned}$$

b)

$$\begin{aligned}\mathsf{P}\big[\,x_T \le 1 \,\wedge\, \max_{t\in[0,T]} x_t \le 2\,\big] \;&=\; N\big(\tfrac{1}{\sqrt{100}}\big) + N\big(\tfrac{2\times 2-1}{\sqrt{100}}\big) - 1 \\ &=\; N(0.1) + N(0.3) - 1 \;\approx\; 0.16\end{aligned}$$

c)

$$\begin{aligned}\mathsf{P}\big[\,x_T \ge -3 \,\wedge\, \min_{t\in[0,T]} x_t \ge -6\,\big] \;&=\; \mathsf{P}\big[\,-x_T \le 3 \,\wedge\, -\min_{t\in[0,T]} x_t \le 6\,\big] \\ &=\; \mathsf{P}\big[\,-x_T \le 3 \,\wedge\, \max_{t\in[0,T]}\{-x_t\} \le 6\,\big] \\ &=\; \mathsf{P}\big[\,x_T \le 3 \,\wedge\, \max_{t\in[0,T]}\{x_t\} \le 6\,\big] \\ &=\; N\big(\tfrac{3}{\sqrt{9}}\big) + N\big(\tfrac{2\times 6-3}{\sqrt{9}}\big) - 1 \\ &=\; N(1) + N(3) - 1 \;\approx\; 0.84\,.\end{aligned}$$

2.Aufgabe: Bezeichnen wir wie üblich das Wiener-Maß mit dW, dann ist

$$\mathsf{Prob}\big[\;x_T \;\ge\; a\;\big] \;=\; \int \chi(\,x_T \;\ge\; a\,)\,dW \;=\; \mathsf{E}\big[\;\chi(\,x_T \;\ge\; a\,)\;\big]$$

Die allgemeine Formel (2) aus dem Loesung6.pdf lautet:

$$\mathsf{E}\big[F(x_t)\big] \;=\; \int_{\mathbb{R}} F(\,\sqrt{t}\,v\,)\,e^{-\frac{v^2}{2}}\,\tfrac{dv}{\sqrt{2\pi}}$$

In unserem Fall ist $t = T$ und

$$F(x_T) \;\; = \;\; \chi(\, x_T \,\geq\, a\,)$$

also bekommen wir

$$\begin{aligned}
\mathsf{Prob}\big[\, x_T \,\geq\, a \,\big] \;\; &= \;\; \mathsf{E}\big[\, \chi(\, x_T \,\geq\, a\,)\,\big] \\
&= \;\; \int_{\mathbb{R}} F(\,\sqrt{T}\, v\,)\, e^{-\frac{v^2}{2}}\, \tfrac{dv}{\sqrt{2\pi}} \\
&= \;\; \int_{\mathbb{R}} \chi(\,\sqrt{T}\, v \,\geq\, a\,)\, e^{-\frac{v^2}{2}}\, \tfrac{dv}{\sqrt{2\pi}} \\
&= \;\; \int_{\mathbb{R}} \chi(\, v \,\geq\, a/\sqrt{T}\,)\, e^{-\frac{v^2}{2}}\, \tfrac{dv}{\sqrt{2\pi}} \\
&= \;\; \int_{a/\sqrt{T}}^{+\infty} e^{-\frac{v^2}{2}}\, \tfrac{dv}{\sqrt{2\pi}} \\
&= \;\; \int_{-\infty}^{+\infty} e^{-\frac{v^2}{2}}\, \tfrac{dv}{\sqrt{2\pi}} \;\; - \;\; \int_{-\infty}^{a/\sqrt{T}} e^{-\frac{v^2}{2}}\, \tfrac{dv}{\sqrt{2\pi}} \\
&= \;\; 1 \, - \, N\big(\, a/\sqrt{T}\,\big) \;\; .
\end{aligned}$$