

Lösungen Blatt 11
Finanzmathematik I

Aufgabe 1: a) We have

$$\begin{aligned} H_{\text{call,perf}}(S_T) &= \max\{ S_T/S_0 - 1, 0 \} \\ &= \frac{1}{S_0} \max\{ S_T - S_0, 0 \} \\ &=: \frac{1}{S_0} H_{\text{call,abs}}(S_T) \end{aligned}$$

where $H_{\text{call,abs}}(S_T) = H_{\text{call}}(S_T)$ is just a standard call in absolute amount to which the Black-Scholes formula can be applied. Thus, since $K = S_0$ and $r = 0$,

$$\begin{aligned} V_0 &= \text{price}(H_{\text{call,perf}}) \\ &= \frac{1}{S_0} \text{price}(H_{\text{call,abs}}) \\ &= \frac{1}{S_0} \{ S_0 N(d_+) - S_0 N(d_-) \} \\ &= N(d_+) - N(d_-) \end{aligned}$$

with

$$\begin{aligned} d_{\pm} &= \frac{\log[\frac{S_0}{S_0}] + (0 \pm \sigma^2/2)T}{\sigma\sqrt{T}} \\ &= \pm \frac{\sigma\sqrt{T}}{2}. \end{aligned}$$

This proves part (a).

b) Using (a), we can write

$$\begin{aligned} V_0 &= N(\sigma\sqrt{T}/2) - N(-\sigma\sqrt{T}/2) \\ &= \frac{N(\sigma\sqrt{T}/2) - N(-\sigma\sqrt{T}/2)}{\sigma\sqrt{T}/2 - (-\sigma\sqrt{T}/2)} \times \{ \sigma\sqrt{T}/2 - (-\sigma\sqrt{T}/2) \} \\ &\approx N'(0) \times \sigma\sqrt{T} \end{aligned}$$

Since

$$N(x) = \int_{-\infty}^x e^{-\frac{y^2}{2}} \frac{dy}{\sqrt{2\pi}}$$

we have

$$N'(x) = e^{-\frac{x^2}{2}} \frac{1}{\sqrt{2\pi}}$$

and we obtain

$$\begin{aligned} V_0 &\approx \frac{1}{\sqrt{2\pi}} e^{-\frac{0^2}{2}} \times \sigma \sqrt{T} \\ &= \frac{1}{\sqrt{2\pi}} \times \sigma \sqrt{T} \\ &= 0.398942.. \times \sigma \sqrt{T} \\ &\approx 0.4 \times \sigma \sqrt{T}. \end{aligned}$$