

Lösungen Übungsblatt 4
Finanzmathematik I

Aufgabe 1: If the price process $\{S_k\}_{k=0}^N$ should be a martingale, we must have, in case of zero interest rates $r = 0$,

$$\mathbb{E}[S_k \mid \{S_j\}_{j=0}^{k-1}] = S_{k-1}$$

for all k . Now it is

$$\begin{aligned} \mathbb{E}[S_k \mid \{S_j\}_{j=0}^{k-1}] &= \mathbb{E}[S_{k-1}(1 + \text{ret}_k) \mid \{S_j\}_{j=0}^{k-1}] \\ &= S_{k-1}(1 + \mathbb{E}[\text{ret}_k]) \\ &\stackrel{!}{=} S_{k-1} \end{aligned}$$

which is the case if

$$\begin{aligned} 0 &\stackrel{!}{=} \mathbb{E}[\text{ret}_k] \\ &= q \times p + \text{ret}_{\text{down}} \times (1 - p) \end{aligned}$$

which gives

$$\text{ret}_{\text{down}} = -\frac{p}{1-p} q.$$

Thus, for $p = 3/4$ we get $\text{ret}_{\text{down}} = -\frac{3/4}{1/4} q = -3q$.

Aufgabe 2: siehe Excel-Sheet