

Lösungen Übungsblatt 1 Finanzmathematik I

Aufgabe 1: We set up a portfolio with a cash amount and some units of the underlying. We let V_t denote the portfolio value at time t . Here, we only have two relevant times: start time $t = 0$ with portfolio value V_0 and maturity T with portfolio value V_T . The initial value of the portfolio is equal to the option price which the option seller receives from the option buyer, thus V_0 is our first variable. Then immediately after the option has been sold, the option seller buys δ units of the underlying, still at start time. Thus, at time $t = 0$ we have the following position:

$$V_0 = \underbrace{V_0 - \delta S_0}_{\text{cash}} + \underbrace{\delta S_0}_{\text{stock}}$$

Then time evolves from $t = 0$ to $t = T$. The cash amount has not changed (if we assume that there are no interest rates, otherwise an interest rate has to be applied to the cash part), but the stock part has changed in value, since the underlying price is now S_T and no longer S_0 . Thus, the portfolio value at time $t = T$ is given by

$$\begin{aligned} V_T &= \underbrace{V_0 - \delta S_0}_{\text{cash}} + \underbrace{\delta S_T}_{\text{stock}} \\ &= V_0 + \delta(S_T - S_0) \end{aligned}$$

with $S_T \in \{110, 80\}$. Since the portfolio value at maturity should be equal to the option payoff, we get the following two equations:

$$\begin{aligned} V_0 + \delta(110 - 100) &= 0 = H(S_T = 110) = \max\{100 - 110, 0\} \\ V_0 + \delta(80 - 100) &= 20 = H(S_T = 80) = \max\{100 - 80, 0\} \end{aligned}$$

or

$$\begin{aligned} V_0 + 10\delta &= 0 \\ V_0 - 20\delta &= 20 \end{aligned}$$

which gives

$$2V_0 + V_0 = 2 \times 0 + 20 \quad \Rightarrow \quad V_0 = 20/3$$

and

$$\delta = -V_0/10 = -2/3.$$