The following slides are taken from the introduction to the course

Linear Programming and Convex Analysis

by Marco Cuturi

Flow of packets in Networks

We follow with an example in networks:

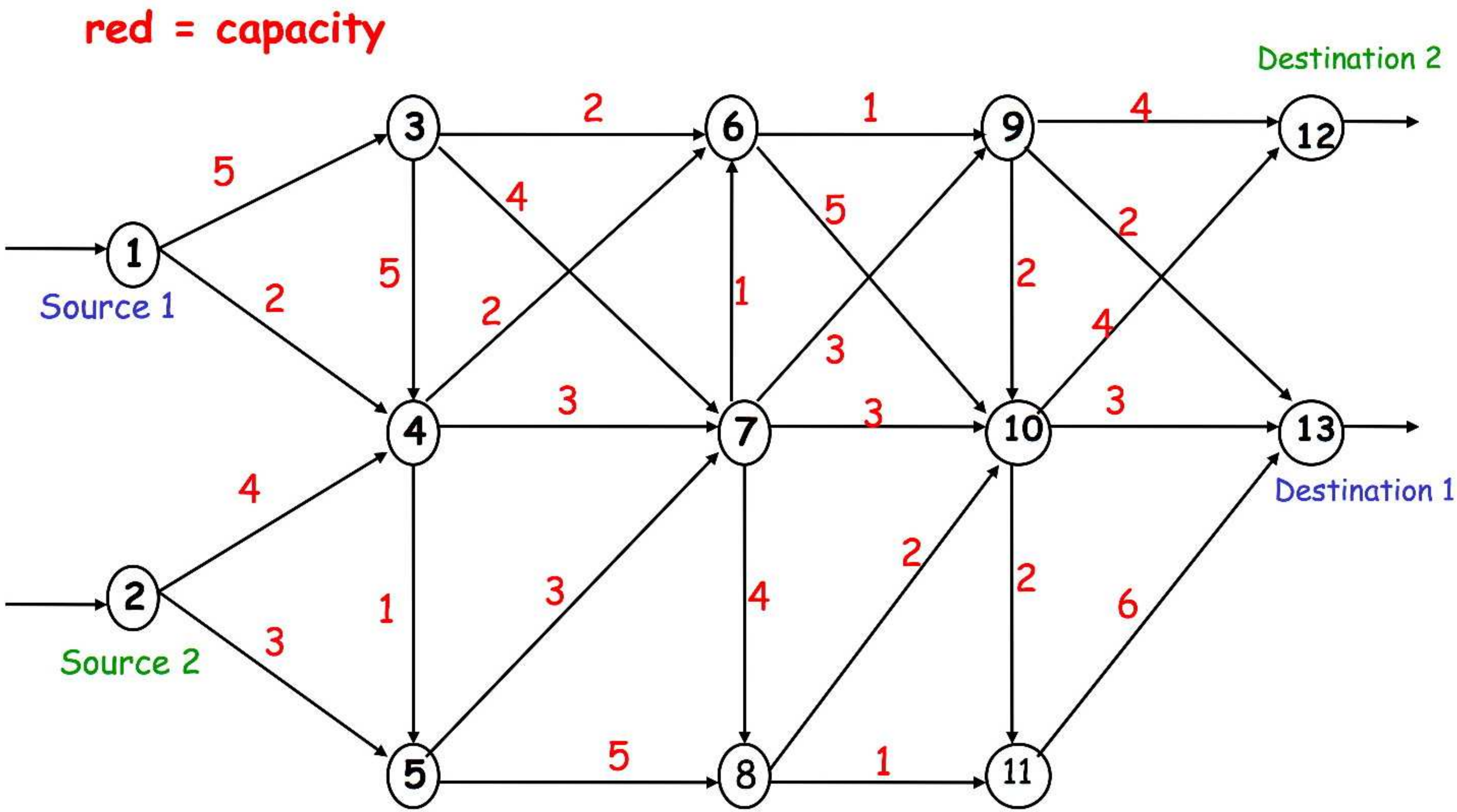
- We use the internet here, but this could be any network (factory floor, transportation, etc).
- Transport data packets from a source to a destination.
- For simplicity: two sources, two destinations.
- Each link in the network has a fixed capacity (bandwidth), shared by all the packets in the network.

Networks: Routing

- When a link is saturated (congestion), packets are simply dropped.
- Packets are dropped at random from those coming through the link.
- Objective: choose a routing algorithm to maximize the **total bandwidth** of the network.

This randomization is not a simplification. TCP/IP, the protocol behind the internet, works according to similar principles.. . .

Networks: Routing



Networks: Routing

A model for the network routing problem: let $N = \{1, 2, \ldots, 13\}$ be the set of network nodes and $L = \{(1,3), \ldots, (11,13)\}$ the set of links.

Variables:

- x_{ij} the flow of packets with origin 1 and destination 1, going through the link between nodes i and j.
- y_{ij} the flow of packets with origin 2 and destination 2, going through the link between nodes i and j.

Parameters:

- u_{ij} the maximum capacity of the link between nodes i and j.

Routing problem: Modeling

Write this as an optimization problem.

Consistency constraints:

- Flow coming out of a node must be less than incoming flow:

$$\sum_{j:\ (i,j)\in L} x_{ij} \leq \sum_{j:\ (j,i)\in L} x_{ij}, \quad \text{for all nodes } i$$

  and

$$\sum_{j:\ (i,j)\in L} y_{ij} \leq \sum_{j:\ (j,i)\in L} y_{ij}, \quad \text{for all nodes } i$$

- Flow has to be positive:

$$x_{ij}, y_{ij} \geq 0, \quad \text{for all } (i,j) \in L$$

Routing problem: Modeling

Capacity constraints:

- Total flow through a link must be less than capacity:

$$x_{ij} + y_{ij} \leq u_{ij}, \quad \text{for all } (i,j) \in L$$

- No packets originate from wrong source:

$$x_{2,4},\ x_{2,5},\ y_{1,3},\ y_{1,4} = 0$$

Objective:

- Maximize total throughput at destinations:

$$\text{maximize } x_{9,13} + x_{10,13} + x_{11,13} + y_{9,12} + y_{10,12}$$

Routing problem: Modelling

The final program is written:

$$
\begin{array}{ll}
\text{maximize} & x_{9,13} + x_{10,13} + x_{11,13} + y_{9,12} + y_{10,12} \\
\text{subject to} & \displaystyle\sum_{j:\ (i,j)\in L} x_{ij} \leq \sum_{j:\ (j,i)\in L} x_{ij} \\
& \displaystyle\sum_{j:\ (i,j)\in L} y_{ij} \leq \sum_{j:\ (j,i)\in L} y_{ij} \\
& x_{ij} + y_{ij} \leq u_{ij} \\
& x_{2,4}, x_{2,5}, y_{1,3}, y_{1,4} = 0 \\
& x_{ij}, y_{ij} \geq 0, \quad \text{for all } (i,j) \in L
\end{array}
$$

Constraints and objective are linear: this is a **linear program**.